# State Feedback Controller for  a Grid Connected Inverter

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**Linear Systems**

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# Abstract

The main focus of this report is control and analyze the stability of the Voltage Source Grid-Connected Inverter with LCL Filter. To connect this inverter to the grid, the inverter current should track the grid current very closely to prevent faults or high currents. Furthermore, the LCL Filter is naturally resonating which outputs a response with high peak overshoot. A full state feedback controller with an observer is used to track the current and, at the same time, provide dampening to the system to reduce the peak overshoot.

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## Introduction

A full bridge DC to AC inverter is used to convert the DC power from PV or Wind to AC as shown in Figure 1. This process does not output a pure sine wave but it outputs a sine wave with harmonics.

LCL Filters are used instead of L or LC Filters because they are better at filtering harmonics. The output of the filter is connected to the AC Grid. A control system is used to track the AC current with the filter output.

## Plant Diagram

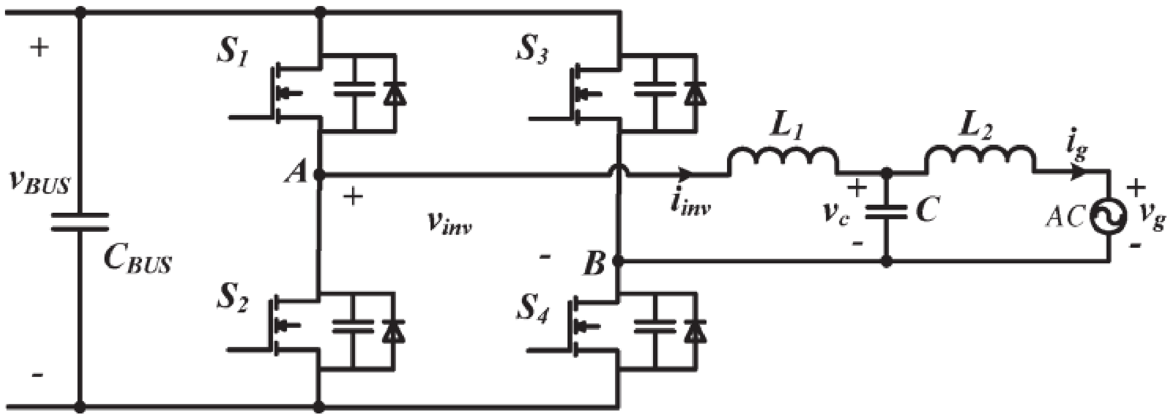



FIGURE 1 MAIN DIAGRAM

This diagram is a model of the voltage source inverter with an DC input and a filtered output. The switches are part of a full bridge rectifier which outputs an ac voltage. This model is simplified by neglecting the switches as shown in Figure 2.

## Simplified Model

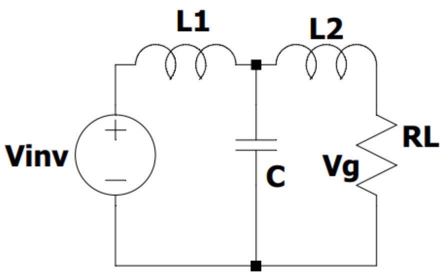



FIGURE 2 SIMPLIFIED MODEL

Using Mesh Analysis on Figure 2 gives us the following differential equations:

$$\frac{di_{inv}}{dt} = \frac{1}{L_1}v_{inv} - \frac{1}{L_1}v_c \text{ — } ①$$

$$\frac{dv_c}{dt} = \frac{1}{C}i_{inv} - \frac{1}{C}ig \text{ — } ②$$

$$\frac{di_g}{dt} = \frac{1}{L_2}v_c - \frac{1}{L_2}v_g \text{ — } ③$$

To convert these differential equations into canonical form the following state variables are selected:

$$x_1 = i_{inv}$$
$$x_2 = v_c$$
$$x_3 = i_g$$

Then the system can be represented by the following equation.

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{L_1} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} \\ 0 & \frac{1}{L_2} & -\frac{R_L}{L_2} \end{bmatrix} x + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} u$$

To get the values of $L_1$, $L_2$, C and $R_L$, the following tables are used from [1].

**TABLE 1**

| Symbol | Parameter | Value |
|---|---|---|
| $Po$ | Output Power | 1KW |
| $V_{dc}$ | Input Voltage | 235-450VDC |
| $V_{ac}$ | Output Voltage | 85-264VAC |
| $f_{sw}$ | Switching frequency | 20KHz |
| $I_{in(max)}$ | Maximum input current | 16A |

**TABLE 2**

| Symbol | Parameter | Value |
|---|---|---|
| $L_1$ | Inverter-side inductor | 1.5mH |
| $L_2$ | Grid-side inductor | 270uH |
| $C$ | LCL-filter capacitor | 10uF |
| $C_{BUS}$ | DC-bus capacitor | 800uH |

Note: $R_L$ is taken as 117.3 Ω which would give us:

$$A = \begin{bmatrix} 0 & -666.7 & 0 \\ 10000 & 0 & -10000 \\ 0 & 3703.7 & -43444.4 \end{bmatrix}$$

$$B = \begin{bmatrix} 666.7 \\ 0 \\ 0 \end{bmatrix}$$

## Control Objective

- To track the grid current to synchronize the inverter output with the grid.
- To provide damping for the naturally resonating LCL filter.

## List of Assumptions

- No switch Losses
- The pairs of switches operate simultaneously
- No power losses
- Ideal LCL Filter
- Constant grid frequency
- Taking the state variables as $i_{inv}$, $v_c$ and $i_g$
- Taking $y = x_3 = i_g$ as output, which is equivalent to:

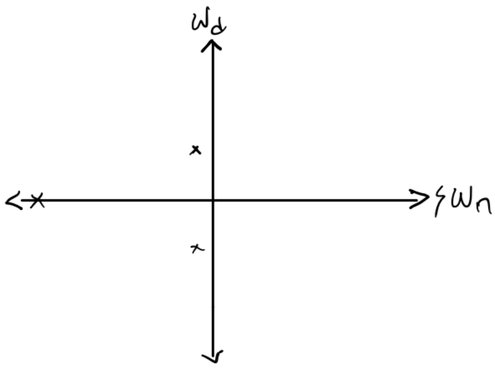
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad D = [0]$$

## Stability

$$A = \begin{bmatrix} 0 & -666.7 & 0 \\ 10000 & 0 & -10000 \\ 0 & 3703.7 & -43444.4 \end{bmatrix}$$

Poles:

```
-433.34 +      2571.9i
-433.34 -      2571.9i
 -42578 +           0i
```




All the poles or eigenvalues are negative or they lie in the left-hand side of the plane as shown in the pole-zero plot above. Thus, the given system is asymptotically stable and therefore, also BIBO stable.

## Controllability

The controllability of a given system can be checked by constructing the controllability matrix using the given equation and determining the rank of the controllability matrix.

$$\mathcal{C} = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}$$

Using MATLAB, the controllability matrix is:

| | | |
|---|---|---|
| 666.7 | 0 | -4.4449e+09 |
| 0 | 6.667e+06 | 0 |
| 0 | 0 | 2.4693e+10 |

s

The controllability matrix is full rank which tells us that the system is fully controllable.

## Observability

The observability of a system can be checked by using the observability matrix which can be found by using the following equation.

$$\mathcal{O}_v = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{v-1} \end{bmatrix}.$$

The observability matrix is:

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | -666.7 | 0 |
| -6.667e+06 | 0 | 6.667e+06 |

The matrix is full rank which shows that the system is fully observable.

## Design of the State Feedback Controller

Parameters from the state space span by the (A, B, C, D) Matrix output the step response in Figure 3: %OP = 223% and Ts = 0.0108 (Very High Overshoot).

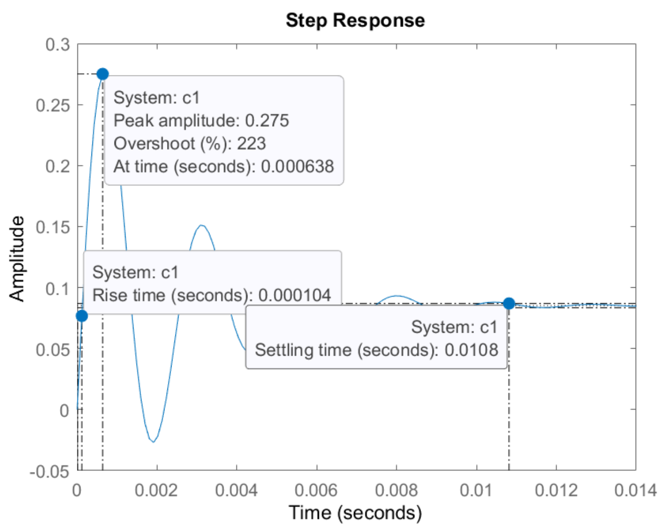



FIGURE 3 ORIGINAL STEP RESPONSE

Let %OP = 10% and Ts = 0.00212. Using the following formulas.

$$PO = \left[ e^{\left(-\pi\zeta/\sqrt{1-\zeta^2}\right)} \right] \cdot 100\%$$

$$T_s = \frac{4}{\zeta\omega_n}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

To get Damping Factor ($\zeta$)= 0.85 & Natural Frequency ($\omega_n$) = 2219.8, which gives us two poles, and we can add the third pole at least 10 points further to minimize its effect on the system.

```
p1 = -1886.8+1170j;
p2 = -1886.8-1170j;
p3 = -2000;
```

Using the Ackermann's Formula, we can get the value of K

```
-56.503        240.79        -2779.3
```

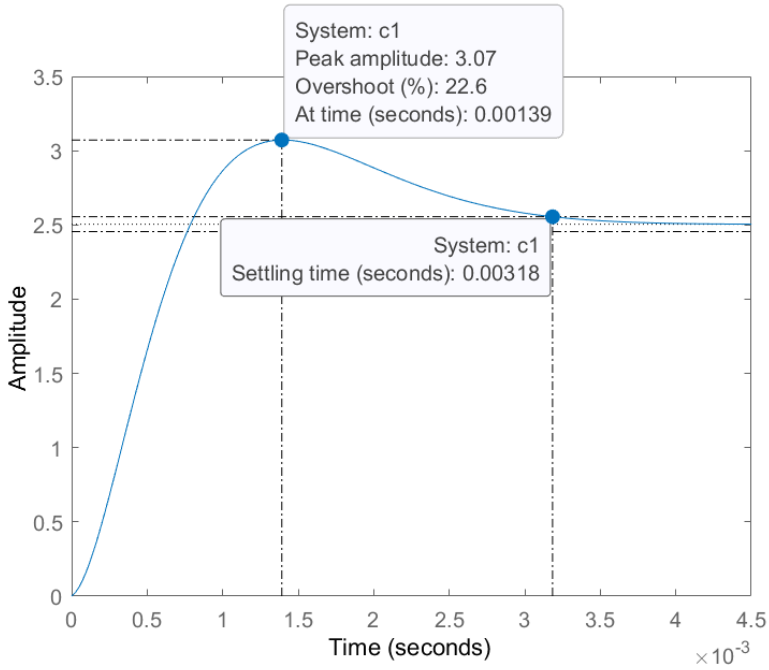



FIGURE 4 STEP RESPONSE WITH FEEDBACK

## Design of the Observer

Choosing Damping Factor ($\zeta$) as 0.8 and Natural Frequency ($\omega_n$) as 10 and taking the third pole 10 points further from the other poles which give us:

```
op1 = -8+6j;
op2 = -8-6j;
op3 = -18;
```

Using Ackermann's Formula, to find the value of L = [-6.1203 -11800 -43410].

The Step Response after adding the observer poles to the full state feedback control is shown in figure 5. The observer poles doesn't affect the system's step response.

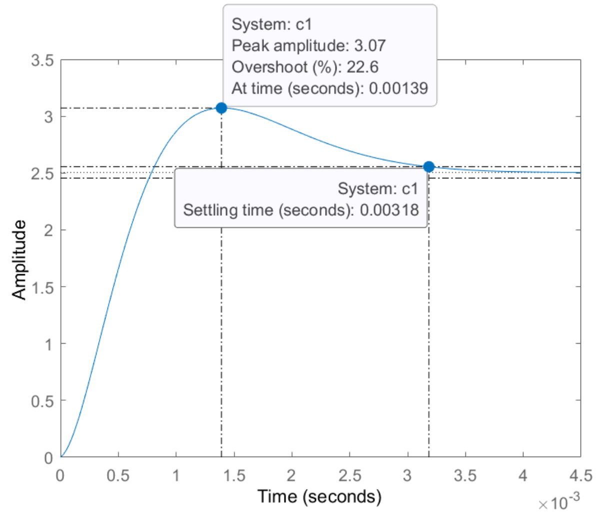



FIGURE 5 STEP RESPONSE AFTER OBSERVER POLES

## Block Diagrams

Using the following equations to construct the block diagram.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \qquad u(t) = -\mathbf{K}\mathbf{x}(t)$$

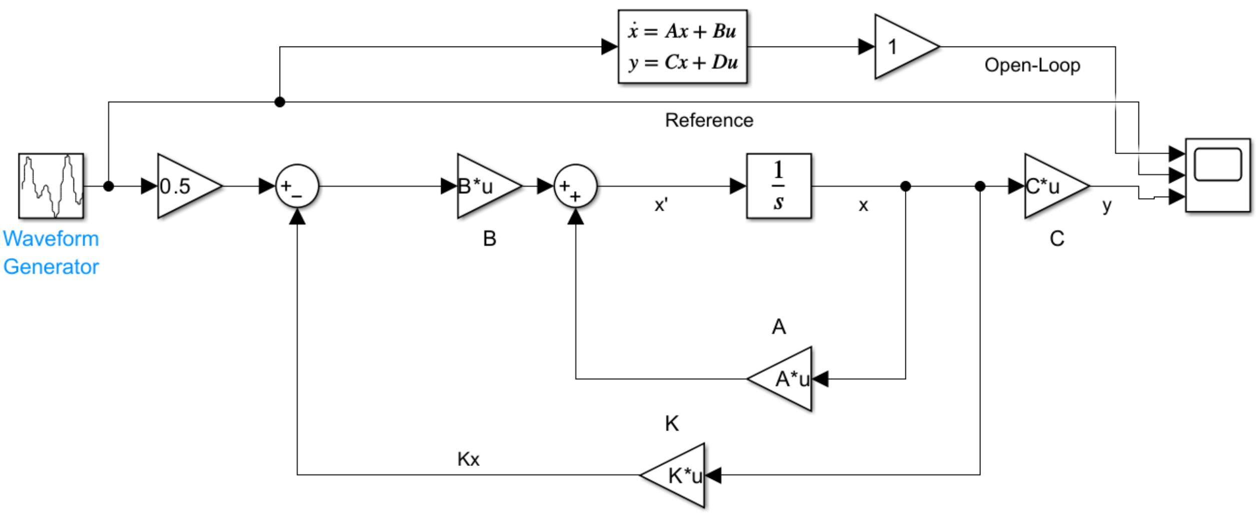



FIGURE 6 FULL STATE FEEDBACK CONTROLLER

Full State Feedback Controller using the K values for the state space system found in this report is shown in figure 6. The waveform generator is simulating the AC grid current. The scope is plotting the reference input, open-loop and closed loop output.

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}u(t) + \mathbf{L}\big(y(t) - \mathbf{C}\hat{\mathbf{x}}(t)\big)$$

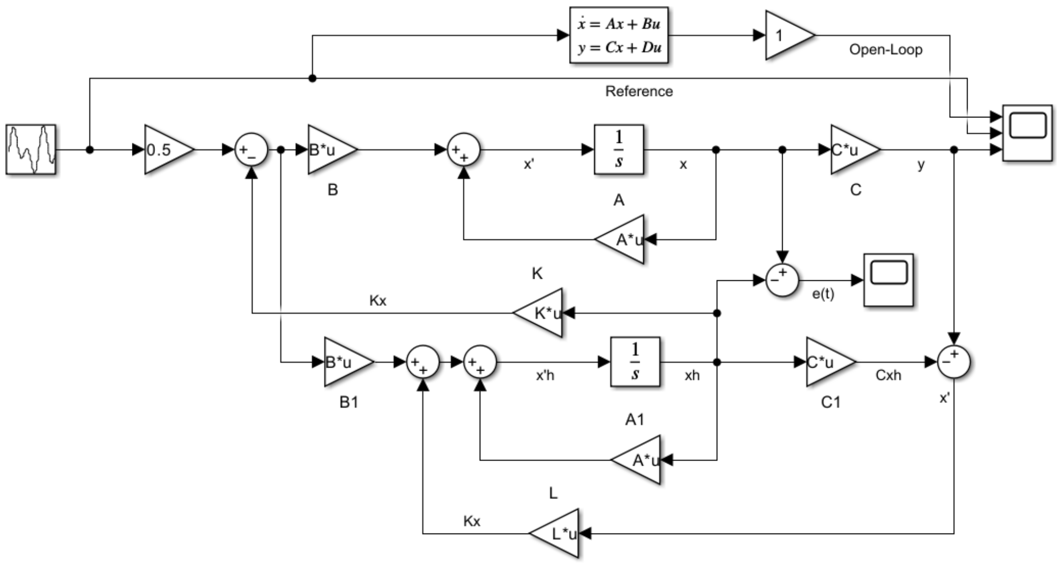
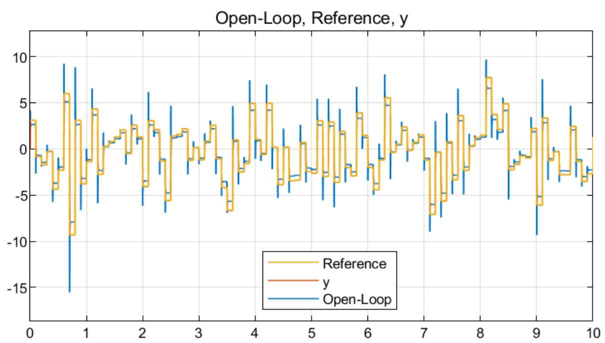


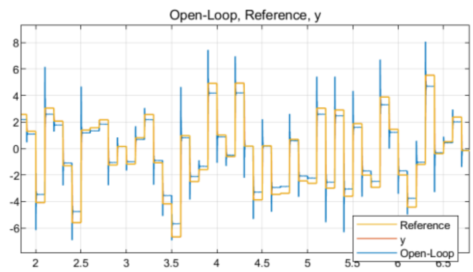

FIGURE 7 FULL STATE FEEDBACK CONTROLLER WITH OBSERVER

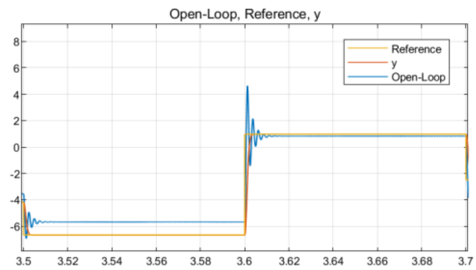
This simulation is very similar to figure 6. The salient feature is that instead of the real values of state variables estimate values are used, which is calculated by the designed observer using L gain values. The waveform generator is simulating the AC grid current. The scope is plotting the reference input, open-loop and closed loop output.

## Simulation Results

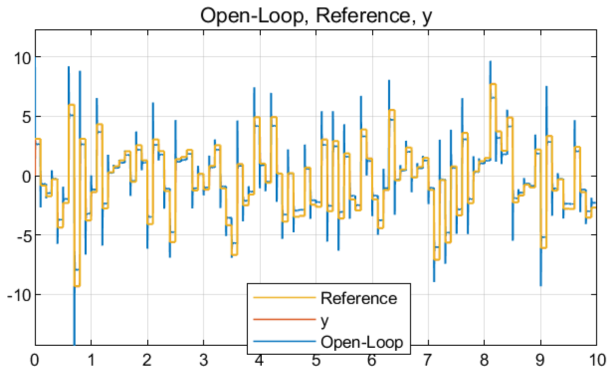
- Full State Feedback Controller output plots which compare the open loop, closed loop and reference. The open loop has resonance, but the closed loop has no resonance when tracking the grid current.

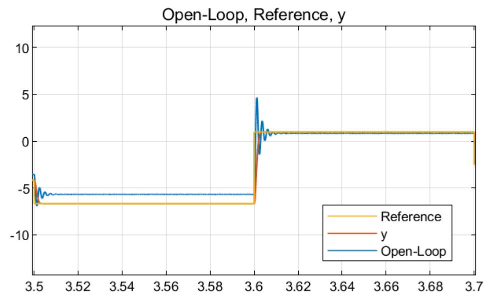


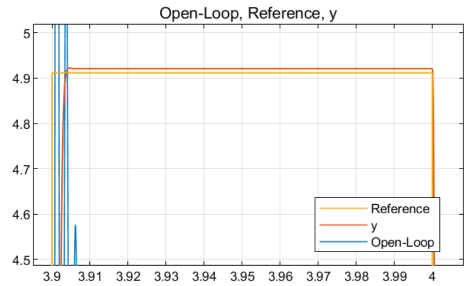



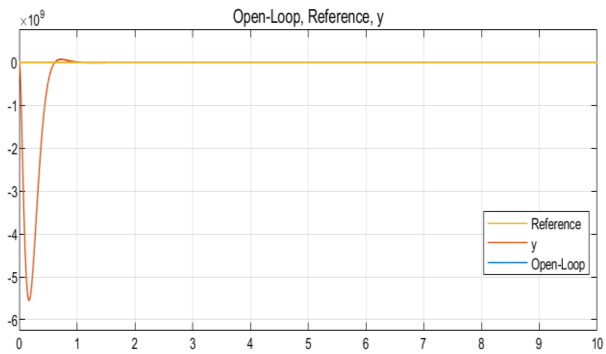
- Using the state observer to get estimate of the states in the full state feedback controller with zero initial conditions. Same results as in the previous simulation.

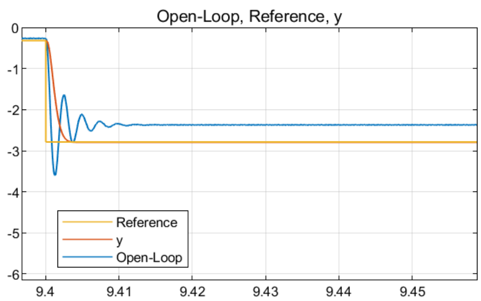


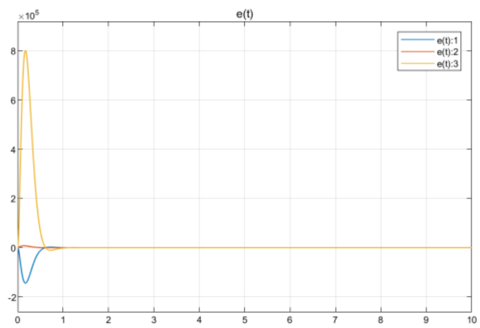




- Using the state observer to get estimate of the states in the full state feedback controller with different initial conditions. This increases the settling time significantly. It takes more than 2 seconds for the output of the loop to settle.



# Discussion of Results

### Full State Feedback Controller (Figure 6)

The open loop response shows resonance in the simulation before settling. Thus, K values were found to reduce this resonance and the output of closed loop which uses the K values is plotted and it shows significant improvement in tracking the reference current.

### Full State Feedback Controller with Observer (Figure 7)

Without initial conditions the output is very similar to the output of figure 6. By using random initial conditions, error between the state estimate and real state values is introduced. The error is very large in the start, this increases the settling time significantly, which is not ideal for connecting to the grid. The state estimates need to be more accurate to fix this error. One way of achieveing a better result is to add a non-linear part to the controller [1].

# Reference

[1] S. Eren, M. Pahlevaninezhad, A. Bakhshai and P. K. Jain, "Composite Nonlinear Feedback Control and Stability Analysis of a Grid-Connected Voltage Source Inverter With LCL Filter," in IEEE Transactions on Industrial Electronics, vol. 60, no. 11, pp. 5059-5074, Nov. 2013, doi: 10.1109/TIE.2012.2225399.

# Appendix

**MATLAB Code**

```
clc
clear all
format short g
A = [ 0 -666.7 0
    10000 0 -10000
     0 3703.7 -43444.4 ]
B = [ 666.7
     0
     0 ]
C = [0 0 1]
D = [0]
plant = ss(A,B,C,D, ...
   'InputName', 'Ig', 'OutputName','Iinv','StateName',{'x1','x2', 'x3'});
poles = pole(plant)
% isstable(plant)
% step(plant);
%right click on the graph to enable characteristics
```

```
CTRB = ctrb(plant)

OBSV = obsv(plant)

rank(CTRB)

rank(OBSV)

%Full State Feedback

%plant.c = [0 0 0; 0 0 0; 0 0 0.3698];

%plant.c = eye(3);

obsv(plant)

%plant.StateName(3) = 'x3';

%plant.OutputName = {'w','i','x3'}

p1 = -1886.8+1170j;

p2 = -1886.8-1170j;

p3 = -2000;

%pole_placement

K = place(A,B,[p1 p2 p3])

% step(plant)

%c1 = feedback(plant,K)

c1 = ss(A-B*K,B,C,0)

poles = pole(c1);

% step(c1)

%K = acker(A,B,[p1 p2 p3]) same as place but numerical

%pole_placement observer

op1 = -8+6j;

op2 = -8-6j;

op3 = -18;

L = place(A',C',[op1 op2 op3])'

At = [ A-B*K            B*K

      zeros(size(A))    A-L*C ];

Bt = [    B

      zeros(size(B)) ];

Ct = [ C    zeros(size(C)) ];

sys = ss(At,Bt,Ct,0)

% step(sys)

pole(sys)
```