Dynamic Modeling of Five-Phase Induction Machine in Terms of dq Winding

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***Abstract*—This document primarily focuses on the mathematical model of a five-phase induction machine in terms of dq windings. Some parts of this document talk about the three-phase derivation as well so that the similarities can be seen. The mathematical model is then implemented in SIMULINK and the outputs are shown. The aim of this project is develop a working MATLAB Simulink Model for a 2-Level 5-Phase Induction Machine to simulate faults and generate datasets under all operating conditions.**

***Index Terms*—model, arbitrary reference frame, simulink, induction machine**

I. Introduction

Multi-phase drives are gaining popularity in high-power and high-speed operations that require precise control, high efficiency [1], and low electromagnetic interference [2]. Fault diagnosis of multi-phase drives is currently a widely researched topic. These machines can also provide fault tolerance [2], which makes them suitable for critical applications that require high reliability. The simulation model will be used for data collection.

A. Background

In order to ensure the safety of a drive system, it is important to detect any potential faults, determine their type and severity, and predict how much useful life remains before failure occurs. Faults within electrical drives can occur for various reasons, which may lead to issues in the machines that make up the system, some of them are:

- Bearing Faults.
- Open or Short Circuit in Windings or Inverters.
- Insulation or Inter-Turn Faults.
- Broken Bars in Squirrel Cage Induction Machine (IM).

Detecting, diagnosing, and predicting failures related to these faults can be done by analyzing the operating signals, currents, voltages, and fluxes [3]. The mathematical model will be used to simulate these faults and collect data under all operating conditions.

II. Assumptions

- q axis can rotate at any arbitrary angular velocity ω or it can be stationary.
- A positive mmf is a south pole due to the stator currents.
- The windings are connected in wye and they are identical.
- Positive directions of the magnetic axes are selected to achieve counterclockwise rotating air-gap mmf with balanced stator currents of the abc sequence.
- The factor $\frac{2}{5}$ makes the magnitude of the variables equal to those of the two-phase system.
- Voltage equations valid for linear and non-linear magnetic fields.
- Two orthogonal windings such that the torque and flux within the machine can be controlled independently.
- Both windings are sinusoidally distributed with $\sqrt{\frac{5}{2}}N_s$.

III. dq Winding Representation

A. Stator dq Winding Representation

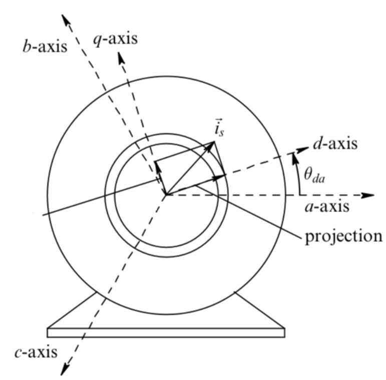

Fig. 1. dq axes in the Stator [4]

Phase currents $i_a(t), i_b(t)$, and $i_c(t)$ are represented by a stator current space vector $\vec{i}_s(t)$. A collinear magnetomotive force (mmf) space vector $\vec{F}_s(t)$ is related to $\vec{i}_s(t)$.

$$\vec{i}_s^a = i_a(t) + i_b(t)e^{j\frac{2\pi}{5}} + i_c(t)e^{j\frac{4\pi}{5}} + i_b(t)e^{j\frac{6\pi}{5}} + i_c(t)e^{j\frac{8\pi}{5}} \tag{1}$$

and

$$\vec{F}_s^a(t) = \frac{N_s}{p}\vec{i}_s^a \tag{2}$$

where N_s is in turns/phase and p is the number of poles.

The currents i_{sd} and i_{sq} can be obtained by equating the mmf produced by the dq windings to that produced by the five-phase windings.

$$\frac{\sqrt{5/2}N_s}{p}(i_{sd} + ji_{sq}) = \frac{N_s}{p}\vec{i}_s^d \tag{3}$$

where the $\vec{i}_s^d$ is expressed using the d-axis as the reference axis.

$$(i_{sd} + ji_{sq}) = \sqrt{\frac{2}{5}}\vec{i}_s^d \tag{4}$$

The factor $\sqrt{2/5}$ ensures that the dq-winding currents produce the same mmf distribution as the five-phase winding currents. Choosing $\sqrt{5/2}N_s$ turns for each of the dq windings results in their magnetizing inductance to be L_m. where L_m is per-phase magnetizing induction for five-phase winding with $i_a + i_b + i_c + i_d + i_e = 0$. Each of these winding has a resistance R_s and a leakage inductance L_{ls}, similar to a-b-c-d-e phase windings.

1) Finding the Factor: For a three-phase machine, calculating flux linkage with phase b using current i_a.

$$L_{mutual} = \frac{\lambda_b}{i_a}|_{(i_b, i_c=0,\text{ rotor open})} \tag{5}$$

Note that only the magnetizing flux (not the leakage flux) produced by i_a links the phase-b winding.

$$\lambda_{b,\text{ due to } i_a} = \cos(\theta_{ab})\lambda_{\text{a, magnetizing due to } i_a} \tag{6}$$

$$= -\frac{1}{2}\lambda_{\text{a, magnetizing due to } i_a} \tag{7}$$

Dividing both sides by i_a to get

$$L_{mutual} = -\frac{1}{2}L_{\text{m,1-phase}} \tag{8}$$

The same mutual inductance exists between phase-a and phase-c, and between phase-b and phase-c.

Under the condition that the rotor is open-circuited, and all three phases are excited such that the sum of the three phase currents is zero.

$$\underset{\text{(rotor, open-circuited)}}{\lambda_{\text{a, magnetizing}}}|_{(i_a+i_b+i_c=0)} = L_{m,1-phase}i_a + L_{mutual}i_b + L_{mutual}i_c \tag{9}$$

Using (8) and replacing $(-i_b - i_c)$ by i_a in (9) To get

$$L_m = \frac{3}{2}L_{m,1-phase} \tag{10}$$

where L_m is the magnetizing inductance per phase. As the induction is proportional to the square of the number of turns, the factor is $\sqrt{\frac{3}{2}}$.

2) Tesla's Rotating Magnetic Field: The angle from the a-axis to the q-axis for a constant angular velocity ω is given by:

$$\theta = \omega t + \theta(0) \tag{11}$$

Relating the rotating position to an adjacent position on stator.

$$\phi_s = \theta + \phi \tag{12}$$

where ϕ_s is the angular displacement about the stator measured from the as axis.
The rotating air-gap magnetomotive force (MMF) in a five-phase induction machine with balanced stator currents can be expressed in terms of the phase currents as follows:

$$mmf_{as} = \frac{N_s}{2}i_{as}\cos(\phi_s) \tag{13}$$

$$mmf_{bs} = \frac{N_s}{2}i_{bs}\cos(\phi_s + \frac{2\pi}{5}) \tag{14}$$

$$mmf_{cs} = \frac{N_s}{2}i_{cs}\cos(\phi_s + \frac{4\pi}{5}) \tag{15}$$

$$mmf_{bs} = \frac{N_s}{2}i_{ds}\cos(\phi_s + \frac{6\pi}{5}) \tag{16}$$

$$mmf_{cs} = \frac{N_s}{2}i_{es}\cos(\phi_s + \frac{8\pi}{5}) \tag{17}$$

$$mmf_s = mmf_{as} + mmf_{bs} + mmf_{cs} + mmf_{ds} + mmf_{es} \tag{18}$$

$$mmf_s = \frac{N_s}{2}[i_{as}\cos(\phi_s) + i_{bs}\cos(\phi_s + \frac{2\pi}{5}) + i_{cs}\cos(\phi_s + \frac{4\pi}{5})] + i_{ds}\cos(\phi_s + \frac{6\pi}{5}) + i_{es}\cos(\phi_s + \frac{8\pi}{5})] \tag{19}$$

For balanced steady-state conditions, the stator currents for an abc sequence may be expressed as

$$i_{as} = \sqrt{2}I_s\cos[\omega_e t + \theta_{esi}(0)] \tag{20}$$

$$i_{bs} = \sqrt{2}I_s\cos[\omega_e t + \frac{2\pi}{5} + \theta_{esi}(0)] \tag{21}$$

$$i_{cs} = \sqrt{2}I_s\cos[\omega_e t + \frac{4\pi}{5} + \theta_{esi}(0)] \tag{22}$$

$$i_{ds} = \sqrt{2}I_s\cos[\omega_e t + \frac{6\pi}{5} + \theta_{esi}(0)] \tag{23}$$

$$i_{es} = \sqrt{2}I_s\cos[\omega_e t + \frac{8\pi}{5} + \theta_{esi}(0)] \tag{24}$$

Substituting the current equations to mmf_s and using (12)

$$mmf_s = \frac{N_s}{2}\sqrt{2}I_s\frac{5}{2}\cos[(\omega_e - \omega)t + \theta_{esi}(0) - \theta(0) - \phi] \tag{25}$$

A $\frac{5}{2}$ factor is added when comparing the mmf_s equation with the common two-phase equation.

B. Rotor dq Windings

Along the same dq axes as in the stator. In the diagram, "A" is for rotor and "a" is for stator

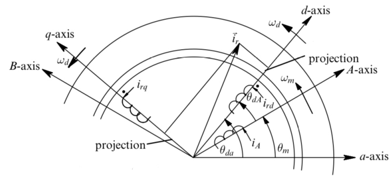

Fig. 2. mmf represented by dq windings [4]

$$\vec{i}_r^A = i_A(t) + i_B(t)e^{j\frac{2\pi}{5}} + i_C(t)e^{j\frac{4\pi}{5}} + i_D(t)e^{j\frac{6\pi}{5}} + i_E(t)e^{j\frac{8\pi}{5}} \tag{26}$$

and

$$\vec{i}_r^A = \frac{\vec{F}_r^A(t)}{N_s/p} \tag{27}$$

The equations are similar to the stator. Each dq winding on the rotor has $\sqrt{5/2}N_s$ turns, and a magnetizing inductance of L_m. Each of these winding has a resistance R_r and a leakage inductance L_{lr}. Mutual inductance between these two windings is zero (orthogonal).

C. Mutual Inductance between dq Windings on the Stator and Rotor

- The mutual inductance between the stator and the rotor d-axis windings is equal to L_m due to the magnetizing flux crossing the air gap.
- Similarly, the mutual inductance between the stator and the rotor q-axis windings equals to L_m
- The mutual inductance between any d-axis winding with any q-axis winding is zero because of their orthogonal orientation, which results in zero mutual magnetic coupling of flux.

IV. Reference Frame Theory

For three-phase transformation (abc sequence) to dq reference. Using the following equations:

$$mmf_s = \frac{N_s}{2}[i_{as}\cos(\phi_s) + i_{bs}\cos(\phi_s - \frac{2\pi}{3}) + i_{cs}\cos(\phi_s + \frac{2\pi}{3})] \tag{28}$$

$$\phi_s = \theta + \phi \tag{29}$$

which yields

$$\begin{aligned} mmf_s = \frac{N_s}{2}\cos(\phi)[i_{as}\cos(\theta) + i_{bs}\cos(\theta - \frac{2\pi}{3}) \\ + i_{cs}\cos(\theta + \frac{2\pi}{3})] - \frac{N_s}{2}\sin(\phi)[i_{as}\sin(\theta) \\ + i_{bs}\sin(\theta - \frac{2\pi}{3}) + i_{cs}\sin(\theta + \frac{2\pi}{3})] \end{aligned} \tag{30}$$

and then we can find i_{ds} ($\phi = 0$) and i_{qs} ($\phi = -\frac{\pi}{2}$) are

$$i_{qs} = i_{as}\cos(\theta) + i_{bs}\cos(\theta - \frac{2\pi}{3}) + i_{cs}\cos(\theta + \frac{2\pi}{3}) \tag{31}$$

$$i_{ds} = i_{as}\sin(\theta) + i_{bs}\sin(\theta - \frac{2\pi}{3}) + i_{cs}\sin(\theta + \frac{2\pi}{3}) \tag{32}$$

To make the transformation matrix, we need to introduce the third variable, which is the neutral current:

$$f_{0s} = \frac{1}{3}(f_{as} + f_{bs} + f_{cs}) \tag{33}$$

And the transformation to the arbitrary reference frame becomes

$$f_{qds} = K_s f_{abcs} \tag{34}$$

$$K_s = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{35}$$

$$(K_s)^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{36}$$

where f can be voltage, current, or flux linkage and $\frac{d\theta}{dt} = \omega(t)$. When deriving for five-phase, we will need to have an extra two equations.

A. Stator Voltage and Flux Linkage

Five-Phase Stator Wye connected. Voltage equations:

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt} \tag{37}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt} \tag{38}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt} \tag{39}$$

$$v_{ds} = r_s i_{ds} + \frac{d\lambda_{ds}}{dt} \tag{40}$$

$$v_{es} = r_s i_{es} + \frac{d\lambda_{es}}{dt} \tag{41}$$

which may be written in matrix notation as

$$v_{abcdes} = r_s i_{abcdes} + \frac{d}{dt}\lambda_{abcdes} \tag{42}$$

from (53), (55) and (42).

$$v_{qd0s} = r_s i_{qd0s} + \omega\lambda_{dq0s} + \frac{d}{dt}\lambda_{dq0s} \tag{43}$$

In expanded form

$$v_{qs} = r_s i_{qs} + \omega\lambda_{qs} + \frac{d}{dt}\lambda_{qs} \tag{44}$$

$$v_{ds} = r_s i_{ds} - \omega\lambda_{ds} + \frac{d}{dt}\lambda_{ds} \tag{45}$$

$$v_{xs} = r_s i_{xs} + \frac{d}{dt}\lambda_{xs} \tag{46}$$

$$v_{ys} = r_s i_{ys} + \frac{d}{dt}\lambda_{ys} \tag{47}$$

$$v_{0s} = r_s i_{0s} + \frac{d}{dt}\lambda_{0s} \tag{48}$$

v_{0s} is zero for balanced conditions and for a wye-connected symmetrical stator. Neglecting the x and y components (d3 and q3 harmonics) to make the analysis simpler.

Let $L_{Ms} = \frac{5}{2}L_{ms}$. Flux linkage equations for a magnetically linear system ($\lambda_{abc} = L_s i_{abc}$) become

$$L_s = (L_{ls} + L_{Ms})I \tag{49}$$

$$\lambda_{qd0s} = L_s i_{qd0s} \tag{50}$$

where I is 3x3 identity. $L_{Ms}i_{0s}$ is coupled between as, bs, and cs axes which sums to zero.

The torque T_e is assumed to be positive in the direction of increasing θ_r whereas the load torque is positive in the opposite direction.

V. Mathematical Relationships of the dq Windings

For Five-Phase to dq-phase transformation matrix, harmonics are used to make an equation [5]. Hereafter, we will drop the superscript to any space vector expressed using the d-axis as the reference.

$$\vec{i}_s(t) = \vec{i}_s^a(t)e^{-j\theta_{da}(t)} \tag{51}$$

Substituting for $\vec{i}_s^a(t)$ from (1)

$$\vec{i}_s(t) = i_a e^{-j\theta_{da}} + i_b e^{-j(\theta_{da}-\frac{2\pi}{5})} + i_c e^{-j(\theta_{da}-\frac{4\pi}{5})} + i_b e^{-j(\theta_{da}-\frac{6\pi}{5})} + i_c e^{-j(\theta_{da}-\frac{8\pi}{5})} \tag{52}$$

Adding 2 equations for third order harmonics in the five phase induction machine (odd). Equating the real and imaginary components on the right side by using $\alpha = \frac{2\pi}{5}$ and (4)
To get $[T_s]_{abcde->dqxy0}$ which is the transformation matrix for the stator. The same transformation matrix relates the stator flux linkages and the stator voltages.

Similarly, replace θ_{da} by θ_{dA} for rotor transformation matrix. $[T_r]_{ABCDE->dqxy0}$ is the transformation matrix for the rotor. Same relationships apply to voltages and flux linkages.

A. Relating dq Winding Variables to Phase Winding Variables

We can add a row at the bottom to represent the condition that all five phase currents sum to zero and use the third order harmonics for the third and fifth row. Then inverting(transpose), orthonormal matrix, the transformation matrix from previous section. where α is $\frac{2\pi}{5}$. A similar transformation matrix $[T_r]_{dqxy0->ABCDE}$ for the rotor can be written by replacing θ_{da} with θ_{dA}. where α is $\frac{2\pi}{5}$.

B. Flux Linkages of dq Windings in Terms of their Currents

Selecting stator d-winding as an example. Due to i_{sd}, both the magnetizing flux as well as the leakage flux link this winding. However, due to i_{rd}, only the magnetizing flux (leakage flux does not cross the air gap) links this stator winding. The transformation matrices are shown in the coming pages.

Writing the flux expressions for all four windings.

Stator Windings:

$$\lambda_{sd} = L_s i_{sd} + L_m i_{rd} \tag{57}$$

and

$$\lambda_{sq} = L_s i_{sq} + L_m i_{rq} \tag{58}$$

where $L_s = L_{ls} + L_m$
Rotor Windings:

$$\lambda_{rd} = L_s i_{rd} + L_m i_{sd} \tag{59}$$

and

$$\lambda_{rq} = L_s i_{rq} + L_m i_{sq} \tag{60}$$

where $L_r = L_{lr} + L_m$

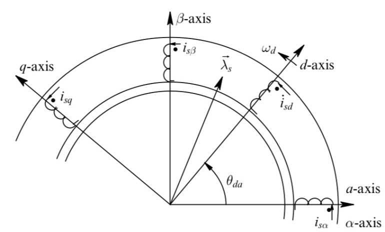

Fig. 3. Stator $\alpha\beta$ and dq equivalent windings

C. dq Winding Voltage Equations

Stator Windings: To derive the dq winding voltages, we will first consider a set of orthogonal $\alpha\beta$ windings affixed to the stator, as shown in Fig. 3-5, where the α-axis is aligned with the stator a-axis. In all windings, the voltage polarity is defined to be positive at the dotted terminal. In $\alpha\beta$ windings in terms of their variables.

$$v_{s\alpha} = R_s i_{s\alpha} + \frac{d\lambda_{s\alpha}}{dt} \tag{61}$$

$$v_{s\beta} = R_s i_{s\beta} + \frac{d\lambda_{s\beta}}{dt} \tag{62}$$

Multiplying (62) by j on both sides. Then adding the result to (61).

$$\vec{v}_{s\alpha\beta}^a = R_s \vec{i}_{s\alpha\beta}^a + \frac{d\vec{\lambda}_{s\alpha\beta}^a}{dt} \tag{63}$$

where $\vec{v}_{s\alpha\beta}^a = v_{s\alpha} + jv_{s\beta}$ and so on. From Figure 3.

$$\vec{v}_{s\alpha\beta}^a = \vec{v}_{sdq}.e^{j\theta_{da}} \tag{64}$$

$$\vec{i}_{s\alpha\beta}^a = \vec{i}_{sdq}.e^{j\theta_{da}} \tag{65}$$

$$\vec{\lambda}_{s\alpha\beta}^a = \vec{\lambda}_{sdq}.e^{j\theta_{da}} \tag{66}$$

where $\vec{v}_{sdq} = v_{sd} + jv_{sq}$ and so on. Using the expressions from Figure 3 into (63). Hence,

$$\vec{v}_{sdq} = R_s \vec{i}_{sdq} + \frac{d\vec{\lambda}_{sdq}}{dt} + j\omega_d \vec{\lambda}_{sdq} \tag{67}$$

$$
\begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \\ i_{sx}(t) \\ i_{sy}(t) \\ i_{s0}(t) \end{bmatrix} = [T_s]_{abcde->dqxy0} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_d(t) \\ i_e(t) \end{bmatrix}
$$

$$
\underbrace{\sqrt{\frac{2}{5}} \begin{bmatrix} \cos(\theta_{da}) & \cos(\theta_{da}-\alpha) & \cos(\theta_{da}-2\alpha) & \cos(\theta_{da}-3\alpha) & \cos(\theta_{da}-4\alpha) \\ -\sin(\theta_{da}) & -\sin(\theta_{da}-\alpha) & -\sin(\theta_{da}-2\alpha) & -\sin(\theta_{da}-3\alpha) & -\sin(\theta_{da}-4\alpha) \\ \cos 3(\theta_{da}) & \cos 3(\theta_{da}-\alpha) & \cos 3(\theta_{da}-2\alpha) & \cos 3(\theta_{da}-3\alpha) & \cos 3(\theta_{da}-4\alpha) \\ -\sin 3(\theta_{da}) & -\sin 3(\theta_{da}-\alpha) & -\sin 3(\theta_{da}-2\alpha) & -\sin 3(\theta_{da}-3\alpha) & -\sin 3(\theta_{da}-4\alpha) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{[T_s]_{abcde->dqxy0}} \tag{53}
$$

$$
\begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \\ i_{rx}(t) \\ i_{ry}(t) \\ i_{r0}(t) \end{bmatrix} = [T_r]_{ABCDE->dqxy0} \begin{bmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \\ i_D(t) \\ i_E(t) \end{bmatrix}
$$

$$
\underbrace{\sqrt{\frac{2}{5}} \begin{bmatrix} \cos(\theta_{dA}) & \cos(\theta_{dA}-\alpha) & \cos(\theta_{dA}-2\alpha) & \cos(\theta_{dA}-3\alpha) & \cos(\theta_{dA}-4\alpha) \\ -\sin(\theta_{dA}) & -\sin(\theta_{dA}-\alpha) & -\sin(\theta_{dA}-2\alpha) & -\sin(\theta_{dA}-3\alpha) & -\sin(\theta_{dA}-4\alpha) \\ \cos 3(\theta_{dA}) & \cos 3(\theta_{dA}-\alpha) & \cos 3(\theta_{dA}-2\alpha) & \cos 3(\theta_{dA}-3\alpha) & \cos 3(\theta_{dA}-4\alpha) \\ -\sin 3(\theta_{dA}) & -\sin 3(\theta_{dA}-\alpha) & -\sin 3(\theta_{dA}-2\alpha) & -\sin 3(\theta_{dA}-3\alpha) & -\sin 3(\theta_{dA}-4\alpha) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{[T_s]_{ABCDE->dqxy0}} \tag{54}
$$

$$
\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_d(t) \\ i_e(t) \end{bmatrix} = [T_s]_{dqxy0->abcde} \begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \\ i_{sx}(t) \\ i_{sy}(t) \\ i_{s0}(t) \end{bmatrix}
$$

$$
\underbrace{\sqrt{\frac{2}{5}} \begin{bmatrix} \cos(\theta_{da}) & -\sin(\theta_{da}) & \cos(3\theta_{da}) & -\sin(3\theta_{da}) & \frac{1}{\sqrt{2}} \\ \cos(\alpha-\theta_{da}) & \sin(\alpha-\theta_{da}) & \cos(3\alpha-3\theta_{da}) & \sin(3\alpha-3\theta_{da}) & \frac{1}{\sqrt{2}} \\ \cos(2\alpha-\theta_{da}) & \sin(2\alpha-\theta_{da}) & \cos(6\alpha-3\theta_{da}) & \sin(6\alpha-3\theta_{da}) & \frac{1}{\sqrt{2}} \\ \cos(3\alpha-\theta_{da}) & \sin(3\alpha-\theta_{da}) & \cos(9\alpha-3\theta_{da}) & \sin(9\alpha-3\theta_{da}) & \frac{1}{\sqrt{2}} \\ \cos(4\alpha-\theta_{da}) & \sin(4\alpha-\theta_{da}) & \cos(12\alpha-3\theta_{da}) & \sin(12\alpha-3\theta_{da}) & \frac{1}{\sqrt{2}} \end{bmatrix}}_{[T_s]_{dqxy0->abcde}} \tag{55}
$$

$$
\begin{bmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \\ i_D(t) \\ i_E(t) \end{bmatrix} = [T_r]_{dqxy0->ABCDE} \begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \\ i_{rx}(t) \\ i_{ry}(t) \\ i_{r0}(t) \end{bmatrix}
$$

$$
\underbrace{\sqrt{\frac{2}{5}} \begin{bmatrix} \cos(\theta_{dA}) & -\sin(\theta_{dA}) & \cos(3\theta_{dA}) & -\sin(3\theta_{dA}) & \frac{1}{\sqrt{2}} \\ \cos(\alpha-\theta_{dA}) & \sin(\alpha-\theta_{dA}) & \cos(3\alpha-3\theta_{dA}) & \sin(3\alpha-3\theta_{dA}) & \frac{1}{\sqrt{2}} \\ \cos(2\alpha-\theta_{dA}) & \sin(2\alpha-\theta_{dA}) & \cos(6\alpha-3\theta_{dA}) & \sin(6\alpha-3\theta_{dA}) & \frac{1}{\sqrt{2}} \\ \cos(3\alpha-\theta_{dA}) & \sin(3\alpha-\theta_{dA}) & \cos(9\alpha-3\theta_{dA}) & \sin(9\alpha-3\theta_{dA}) & \frac{1}{\sqrt{2}} \\ \cos(4\alpha-\theta_{dA}) & \sin(4\alpha-\theta_{dA}) & \cos(12\alpha-3\theta_{dA}) & \sin(12\alpha-3\theta_{dA}) & \frac{1}{\sqrt{2}} \end{bmatrix}}_{[T_r]_{dqxy0->ABCDE}} \tag{56}
$$

where ω_d is the instantaneous speed (in electrical radians per second) of the dq winding set in the air gap. Separating real and imaginary components:

$$
v_{sd} = R_s i_{sd} + \frac{d}{dt}\lambda_{sd} - \omega\lambda_{sd} \tag{68}
$$

$$
v_{sq} = R_s i_{sq} + \frac{d}{dt}\lambda_{sq} + \omega\lambda_{sq} \tag{69}
$$

ω_d is the speed of dq reference frame relative to the actual physical stator winding speed. In vector form:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = R_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} + \omega_d \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{M_{rotate}} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} \tag{70}$$

where M_{rotate} corresponds to the operator (j), where $j = e^{j\pi/2}$. It rotates the space vector $\vec{\lambda}_{sdq}$ by an angle of $\pi/2$.

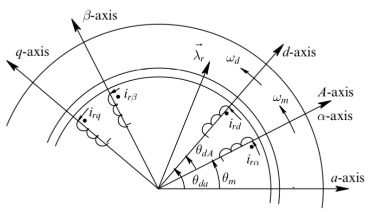

Fig. 4. Rotor $\alpha\beta$ and dq equivalent windings

Rotor Windings: An analysis similar to the stator case. α axis is aligned to the stator A axis. The same d-axis is used as the stator. Replace θ_{da} by θ_{dA}.

$$v_{r\alpha} = R_r i_{s\alpha} + \frac{d\lambda_{r\alpha}}{dt} \tag{71}$$

$$v_{r\beta} = R_r i_{r\beta} + \frac{d\lambda_{r\beta}}{dt} \tag{72}$$

Multiplying (72) by j on both sides. Then adding the result to (71).

$$\vec{v}^{a}_{r\alpha\beta} = R_r \vec{i}^{a}_{r\alpha\beta} + \frac{d\vec{\lambda}^{a}_{r\alpha\beta}}{dt} \tag{73}$$

where $\vec{v}^{a}_{r\alpha\beta} = v_{r\alpha} + jv_{r\beta}$ and so on. From Figure 4.

$$\vec{v}^{a}_{r\alpha\beta} = \vec{v}_{rdq}.e^{j\theta_{dA}} \tag{74}$$

$$\vec{i}^{a}_{r\alpha\beta} = \vec{i}_{rdq}.e^{j\theta_{dA}} \tag{75}$$

$$\vec{\lambda}^{a}_{r\alpha\beta} = \vec{\lambda}_{rdq}.e^{j\theta_{dA}} \tag{76}$$

where $\vec{v}_{rdq} = v_{rd} + jv_{rq}$ and so on. Using the expressions from Figure 4 into (73). Hence,

$$\vec{v}_{rdq} = R_r \vec{i}_{rdq} + \frac{d\vec{\lambda}_{rdq}}{dt} + j\omega_{dA}\vec{\lambda}_{rdq} \tag{77}$$

Separating real and imaginary components of equation (77):

$$v_{rd} = R_r i_{rd} + \frac{d}{dt}\lambda_{rd} - \omega_{dA}\lambda_{rd} \tag{78}$$

$$v_{rq} = R_r i_{rq} + \frac{d}{dt}\lambda_{rq} + \omega_{dA}\lambda_{rq} \tag{79}$$

where ω_{dA} is the instantaneous speed (in electrical radians per second) of the dq winding set in the air gap with respect to the rotor A-axis speed (rotor speed). That is

$$\omega_{dA} = \omega_d - \omega_m \tag{80}$$

ω_m is the rotor speed in electrical radians per second. It is related to w_{mech}, the rotor speed in actual radians per second, by the pole-pairs as follows:

$$\omega_m = (p/2)\omega_{mech} \tag{81}$$

In vector form:

$$\begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix} = R_r \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} + \omega_d \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{M_{rotate}} \begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} \tag{82}$$

where M_{rotate} corresponds to the operator (j), where $j = e^{j\pi/2}$. It rotates the space vector $\vec{\lambda}_{rdq}$ by an angle of $\pi/2$.

D. *Obtaining Fluxes and Currents with Voltages as Inputs*

Writing equations (70) and (82) in a state space form.

$$\frac{d}{dt}\begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} - R_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} - \omega_d \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{M_{rotate}} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} \tag{83}$$

and

$$\frac{d}{dt}\begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} = \begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix} - R_r \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} - \omega_d \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{M_{rotate}} \begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} \tag{84}$$

In vector form

$$\frac{d}{dt}[\lambda_{sdq}] = [v_{sdq}] - R_s[i_{sdq}] - \omega_d[M_{rotate}][\lambda_{sdq}] \tag{85}$$

$$\frac{d}{dt}[\lambda_{rdq}] = [v_{rdq}] - R_r[i_{rdq}] - \omega_d[M_{rotate}][\lambda_{rdq}] \tag{86}$$

E. *Choice of the dq Winding Speed ω_d*

- It is possible to assume any arbitrary value for ω_d.
- Changing the winding equates to changing the transformation (from Clarke to Park), and so on [6].
- $\omega_d = \omega_{syn}$, 0 or ω_m, where ω_m is the speed in synchronous speed in electrical radians per second.
- Corresponding values for ω_{dA} equal ω_{slip}, $-\omega_m$ or 0 respectively, where $\omega_{slip} = \omega_{syn} - \omega_m$ in electrical radians per second.
- Under a balanced steady state, choosing $\omega_d = \omega_{syn}$ results in the dq windings rotating at the same speed as the field distribution in the air gap. Currents, voltages, and flux linkages associtated with the stator and rotor dq windings are dc in a balanced sinusoidal steady state.
- Easy to design PI controllers for dc quantities
- $\omega_d = 0$, a stationary d-axis (often $\theta_{da} = 0$), leads to rotor and stator dq winding voltages and currents oscillating at the synchronous frequency in a balanced sinusoidal steady state.
- $\omega_d = \omega_m$ results in dq winding voltages and currents in the stator and rotor varying at the slip frequency, choice for analyzing synchronous machines.

VI. Electromagnetic Torque

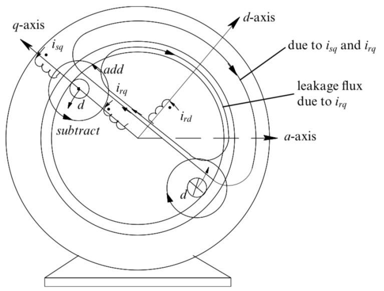

Fig. 5. Torque on the rotor d-axis

A. Torque on the Rotor d-Axis Winding

On the rotor d-axis torque is produced due to the flux density produced by q axis windings in Figure 5.

$$B_{rq}(\theta) = \mu_o H_{rq}(\theta) = (\frac{\mu_o N_s}{pl_g}) i_a \cos(\theta) \tag{87}$$

For peak flux density

$$B_{rq} = \frac{\mu_o}{l_g} \underbrace{(\frac{\sqrt{5/2}N_s}{p})(i_{sq} + \frac{L_r}{L_m} i_{rq})}_{mmf} \tag{88}$$

where the factor L_r/L_m allows us to include both the magnetizing and leakage flux produced by i_rq According to the figure, the torque on the rotor is counterclockwise, so we will consider it positive. Let $x = d3$ and $y = q3$.

$$T_{d,rotor} = \frac{p}{2}(L_m i_{sq} + L_r i_{rq}) i_{rd} = \frac{p}{2} \lambda_{rq} i_{rd} \tag{89}$$

$$\begin{aligned} T_{d_3,rotor} &= 3 * \frac{p}{2}(L_m i_{sq_3} + L_r i_{rq_3}) i_{rd_3} \\ &= 3 * \frac{p}{2} \lambda_{rq_3} i_{rd_3} \end{aligned} \tag{90}$$

B. Torque on the Rotor q-Axis Winding

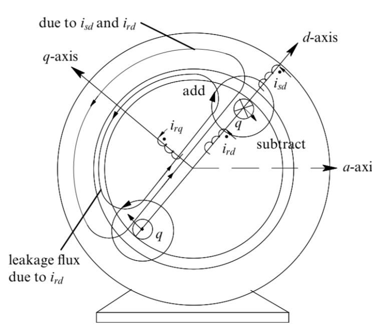

Fig. 6. Torque on the rotor q-axis

On the rotor q-axis winding, the torque produced is due to the flux density produced by the d-axis windings in Figure 6. This is torque is clockwise, so it will be considered as negative. Similar derivation as for rotor d-axis winding.

$$T_{d,rotor} = -\frac{p}{2}(L_m i_{sq} + L_r i_{rq}) i_{rd} = -\frac{p}{2} \lambda_{rq} i_{rd} \tag{91}$$

C. Net Electromagnetic Torque T_{em} on the Rotor

Using Superposition

$$T_{em} = T_{d,rotor} + T_{q,rotor} \tag{92}$$

$$T_{em} = \frac{p}{2}(\lambda_{rq} i_{rd} - \lambda_{rd} i_{rq}) \tag{93}$$

$$T_{em} = \frac{p}{2} L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) \tag{94}$$

$$\begin{aligned} T_{em} = \frac{p}{2} & L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) \\ & + 3 * \frac{p}{2} L_m (i_{sq_3} i_{rd_3} - i_{sd_3} i_{rq_3}) \end{aligned} \tag{95}$$

VII. Electrodynamics

The acceleration is determined by the difference of the electromagnetic torque and the load torque (including friction torque) action on J_{eq}, the combined inertia of the load and the motor.

$$\frac{d}{dt}\omega_{mech} = \frac{T_{em} - T_L}{J_{eq}} \tag{96}$$

VIII. Computer Simulation

Choosing flux linkages as state variables because these quantities change slowly compared with currents, which can almost change instantaneously. Relating flux linkages with current using (57), (58) , (59) and (60).

$$\begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \\ \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} = \underbrace{\begin{bmatrix} L_s & 0 & Lm & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}}_{[M]} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \tag{97}$$

Currrents can be calculated using the inverse of matrix [M]

$$\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} = [M]^{-1} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \\ \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} \tag{98}$$

With voltages as input and choosing the speeed ω_d of the dq windings, the flux linkages are calculated using (85) and (86). The currents are calculated using (98). Combining these with torque equation (91) and the electrodynamics equation (96), we can draw the overall block diagram for computer modelling.

A. Calculation of Initial Conditions

We need to calculate initial values of the state variables (flux linkages of the dq winding). These can be calculated in term of the initial values of the dq winding currents. These currents allow us to computer the electromagnetic torque in steady state. Thus the initial loading of the induction machine.

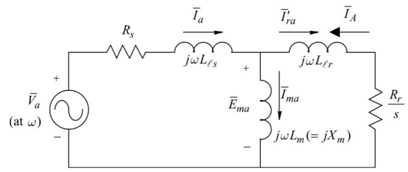

Fig. 7. Per Phase Equivalent Circuit in Steady State

Phasor Analysis: Requires the Per Phase Equivalent Circuit Model

Voltage Equations: Choosing ω_d as ω_{syn}, all dq winding variables are dc quantities and $\omega_{dA} = s\omega_{syn}$. Using equations (68), (69), (78) and (79). Substituting the above equations for flux linkages from equation (57), (58), (59) and (60).
Once the dq winding currents are calculated, the flux linkages can be calculated from (97). The load torque can be calculated using (95). ω_{mech} can be calculated using (96). The results for MATLAB simulation are shown in the next section. Using the following motor parameters for three phase machine:

TABLE I
THREE PHASE INDUCTION MOTOR PARAMETERS

Parameter	Value
R_s (Stator resistance)	1.77 Ω
R_r (Rotor resistance)	1.34 Ω
X_{ls} (Stator leakage reactance)	5.25 Ω
X_{lr} (Rotor leakage reactance)	4.57 Ω
X_m (Magnetizing reactance)	139.0 Ω
Slip (Full Load)	1.72%
P_{il} (Iron Loss)	78 W
P_{ml} (Mechanical Loss)	24 W
J_{eq} (Inertia)	0.025 kg.m^2

This is the architecture of the model used in Simulink.

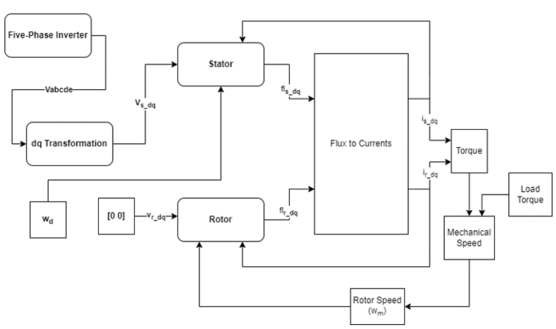

Fig. 8. Block Diagram for Induction Machine on MATLAB

IX. SIMULATION RESULTS

The simulations were performed using the data given in Table 1 on MATLAB Simulink.

A. Three Phase Induction Machine

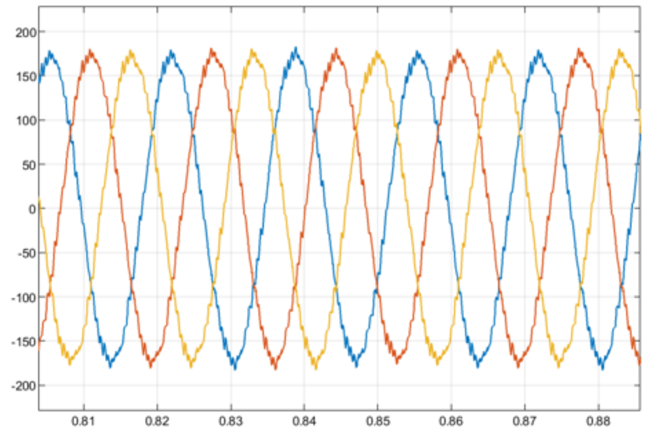

Fig. 9. Three-Phase Inverter Output

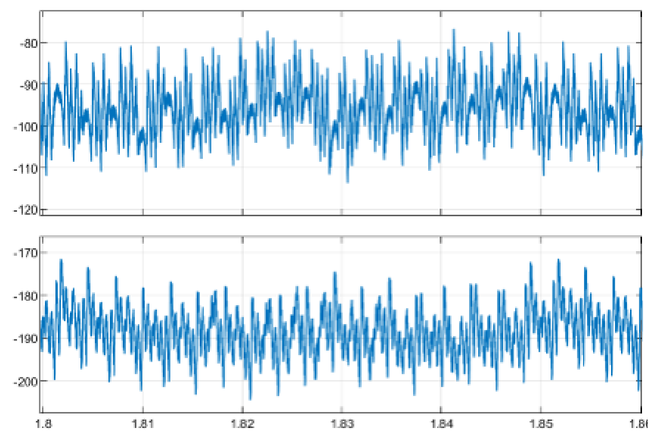

Fig. 10. Stator Voltage in dq

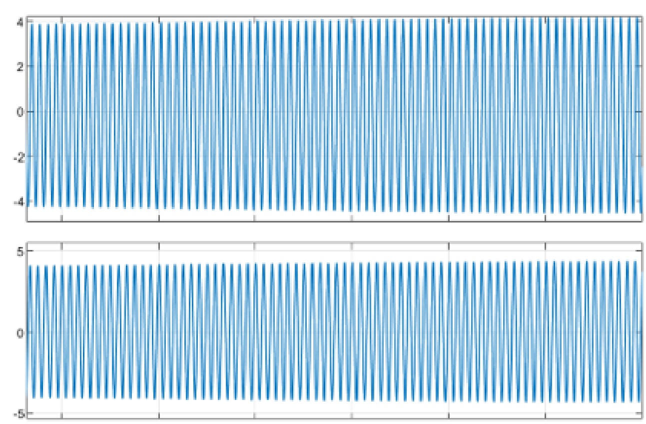

Fig. 11. Stator Flux Linkage in dq

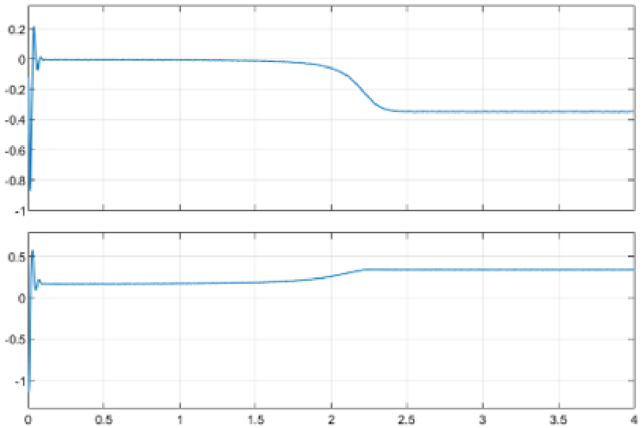

Fig. 12. Rotor Flux Linkage in dq

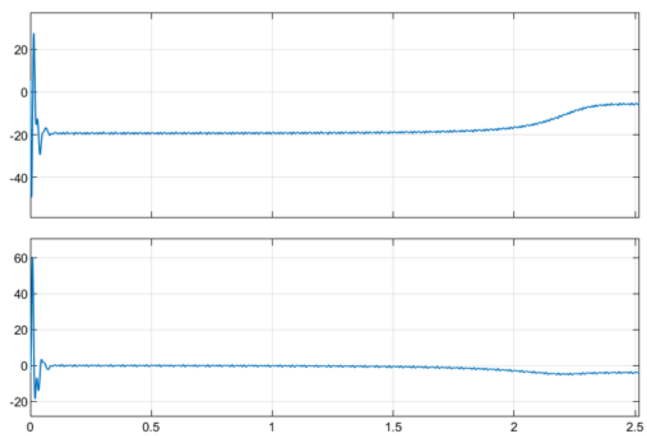

Fig. 13. Stator Current in dq

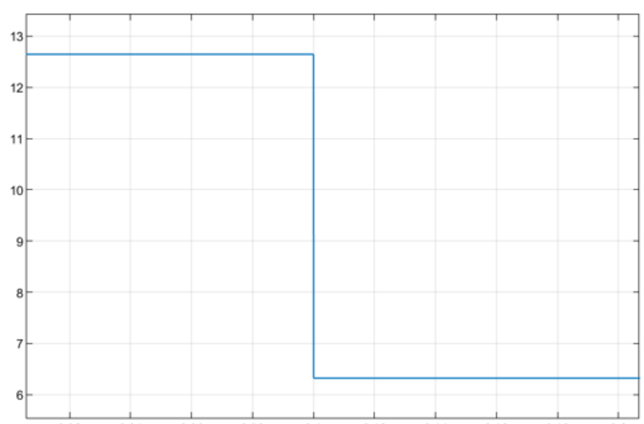

Fig. 14. Load Torque

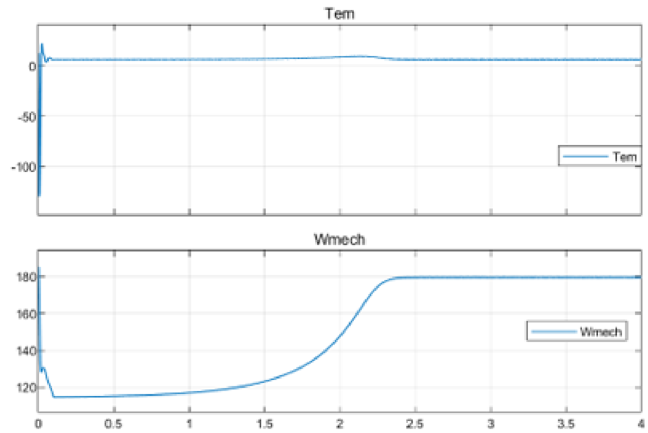

Fig. 15. Torque and Mechanical Speed

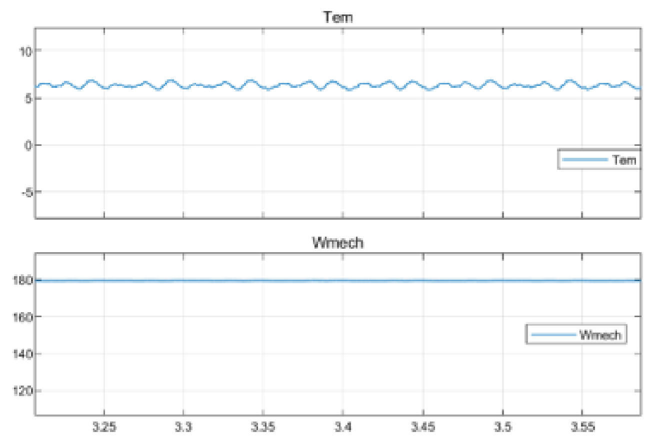

Fig. 16. Torque and Mechanical Speed (Zoomed in)

B. Five-Phase Induction Machine

Using the same data from Table 1. The five-phase induction machine model worked well if the input was a pure sine wave. However, for the filtered inverter output, it could not converge.

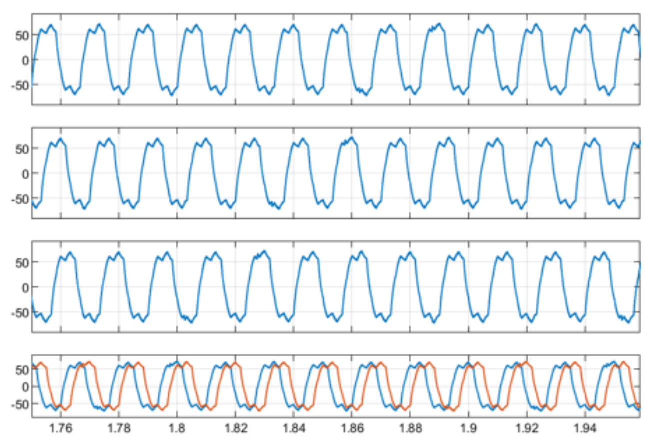

Fig. 17. Five-Phase Inverter Output

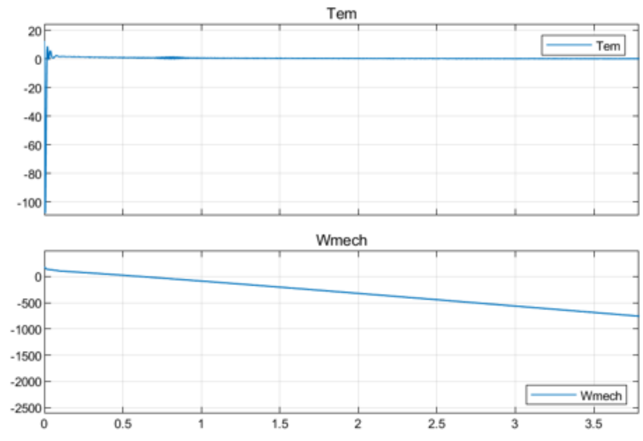

Fig. 18. Using Inverter Output

The mechanical speed keeps on increasing for filtered (not real sine wave) and the T_{em} is almost equal to zero as shown in Fig. 19. However, for input as a real sine wave to the stator gives us a similar output as seen for Three-Phase.

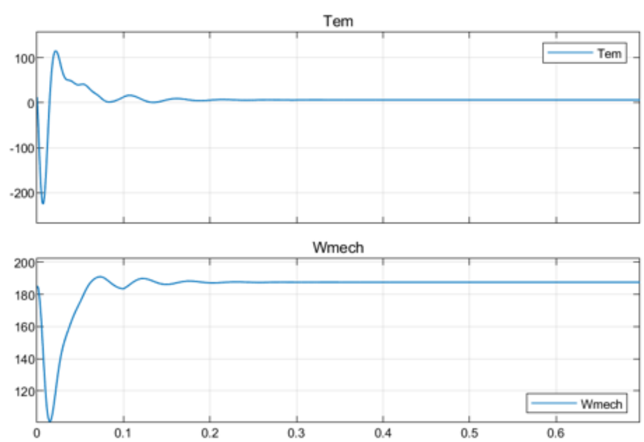

Fig. 19. Using Real Sine Wave

X. Conclusion

The model was tested using existing literature and only the three-phase model closely matched the literature. But the five-phase model only worked when a pure sine wave was given to the stator.

Further, in the above calculations, the third-order harmonics were not considered. The model can be made accurate after the transformation matrix calculations by considering the third-order harmonics.

References

[1] F. Barrero and M. J. Duran, "Recent advances in the design, modeling, and control of multiphase machines - Part I," *IEEE Transactions on Industrial Electronics*, vol. 63, pp. 449–458, 1 2016.

[2] M. J. Duran and F. Barrero, "Recent advances in the design, modeling, and control of multiphase machines - Part II," *IEEE Transactions on Industrial Electronics*, vol. 63, pp. 459–468, 1 2016.

[3] E. Strangas, G. Clerc, H. Razik, and A. Soualhi, *Fault Diagnosis, Prognosis, and Reliability for Electrical Machines and Drives*.

[4] N. Mohan, *Advanced Electric Drives: Analysis, Control, and Modeling Using MATLAB / Simulink*. John Wiley & Sons, 2014.

[5] E. Levi, R. Bojoi, F. Profumo, H. Toliyat, and S. Williamson, "Multiphase induction motor drives - a technology status review," *Electric Power Applications, IET*, vol. 1, pp. 489 – 516, 08 2007.

[6] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, and S. Pekarek, *Analysis of Electric Machinery and Drive Systems*. John Wiley & Sons, 2013.

[7] P. Krause, O. Wasynczuk, and S. Pekarek, *Electromechanical Motion Devices*. Wiley-IEEE Press, 2012.

[8] G. Diyoke and E. U, "Modeling and simulation of five-phase induction motor fed with pulse width modulated five-phase multilevel voltage source inverter topologies," 09 2019.

[9] J. Sun, C. Li, Z. Zheng, and K. Wang, "A generalized, fast and robust open-circuit fault diagnosis technique for star-connected symmetrical multiphase drives," *IEEE Transactions on Energy Conversion*, vol. 37, no. 3, p. 1921–1933, 2022.

[10] U. Mahanta, D. Patnaik, B. Panigrahi, and A. Panda, "Dynamic modeling and simulation of svm-dtc of five phase induction motor," in *2015 International Conference on Energy, Power and Environment: Towards Sustainable Growth (ICEPE)*, p. 1–6, IEEE, 2015.

[11] G. Renukadevi and K. Rajambal, "Generalized model of multi-phase induction motor drive using matlab/simulink," in *2011 IEEE PES Innovative Smart Grid Technologies-India*, p. 1–6, IEEE, 2011.

[12] S. Ahmad and A. Mishra, "Mathematical modelling, simulation and control of five-phase induction motor drives," in *2020 International Conference on Emerging Frontiers in Electrical and Electronic Technologies (ICEFEET)*, p. 1–6, IEEE, 2020.

[13] S. A. Gaikwad and S. Shinde, "Five-phase induction motor modeling and its analysis using matlab/simulink," in *Smart Technologies for Energy, Environment and Sustainable Development*, p. 583–593, Springer, 2022.

[14] S. Suresh and S. S. Dash, "Modeling of five phase induction motor drive," in *2017 International Conference on Technological Advancements in Power and Energy (TAP Energy)*, pp. 1–6, 2017.

[15] R. Kumar and R. K. Saket, "Mathematical modelling, simulation and control of five-phase induction motor drives," in *2020 International Conference on Emerging Trends in Electrical Electronics & Sustainable Energy Systems (ICETEESES)*, pp. 1–6, 2020.